

$$2x^3 e^{x^2}$$



High Level Problem

$$\int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x} dx$$

$$I = \int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x} dx$$

$$= \int \frac{(\sin^4 x)^2 - (\cos^4 x)^2}{1 - 2\sin^2 x \cos^2 x} dx$$

$$= \int \frac{(\sin^4 x - \cos^4 x)(\sin^4 x + \cancel{\cos^4 x})}{\sin^4 x + \cancel{\cos^4 x}} dx$$

$$= \int (\sin^4 x - \cos^4 x) dx$$

$$\boxed{\begin{aligned} \sin^4 x + \cos^4 x &= (\sin^2 x + \cos^2 x)^2 - \\ &\quad (\sin^2 x)^2 \text{ OR } (\cos^2 x)^2 - 2\sin^2 x \cos^2 x \end{aligned}}$$

$$= 1 - 2\sin^2 x \cos^2 x$$

$$= \int [(\sin^2 x)^2 - (\cos^2 x)^2] dx$$

$$= \int \underbrace{(\sin^2 x + \cos^2 x)}_1 (\sin^2 x - \cos^2 x) dx$$

$$= \int (\sin^2 x - \cos^2 x) dx$$

$$= -\int (\cos^2 x - \sin^2 x) dx$$

$$= - \int \cos 2x \, dx$$

$$I = - \frac{\sin 2x}{2} + C$$

$$\int \frac{\cos 5x + \cos 4x}{1 - 2 \cos 3x} dx$$

$$I = \int \frac{\cos 5x + \cos 4x}{1 - 2 \cos 3x} dx$$

$$= \int \frac{2 \cos \frac{9x}{2} \cos \frac{x}{2}}{1 - 2 \left[2 \cos^2 \frac{3x}{2} - 1 \right]}$$

$$= \int \frac{2 \cos \frac{x}{2} \left[4 \cos^3 \left(\frac{3x}{2} \right) - 3 \cos \frac{3x}{2} \right]}{1 - 4 \cos^2 \frac{3x}{2} + 2}$$

$$\cos C + \cos D = 2 \cos \frac{C+D}{2} \cos \frac{C-D}{2}$$

$$\cos x = 2 \cos^2 \frac{x}{2} - 1$$

$$\cos 3x = 4 \cos^3 x - 3 \cos x$$

$$\cos \frac{9x}{2} = \cos 3 \left[\frac{3x}{2} \right]$$

$$\int \frac{2 \cos \frac{x}{2} \cos \frac{3x}{2} \left[4 \cos^2 \frac{3x}{2} - 3 \right]}{3 - 4 \cos^2 \frac{3x}{2}}$$

$$- \int \frac{2 \cos \frac{x}{2} \cos \frac{3x}{2} \left[3 - 4 \cancel{\cos^2 \frac{3x}{2}} \right]}{3 - 4 \cancel{\cos^2 \frac{3x}{2}}}$$

$$= - \int 2 \cos \frac{x}{2} \cos \frac{3x}{2} dx$$

$$= - \left[\int \cos \left(\frac{x}{2} + \frac{3x}{2} \right) + \cos \left(\frac{3x}{2} - \frac{x}{2} \right) dx \right]$$

$$\therefore 2 \cos A \cdot \cos B = \cos(A+B) + \cos(A-B)$$

$$- \left[\int \cos 2x dx + \int \cos x dx \right]$$

$$= - \left[\frac{\sin 2x}{2} + \sin x \right] + c$$

Ans

$$\int \frac{dx}{(x^3 + 3x^2 + 3x + 1)\sqrt{x^2 + 2x - 3}}$$

$$I = \int \frac{dx}{(x+1)^3 \sqrt{x^2 + 2x - 3 + 1 - 1}}$$

$$= \int \frac{dx}{(x+1)^3 \sqrt{(x+1)^2 - 4}}$$

$$= \int \frac{2 \sec \theta \tan \theta \, d\theta}{(2 \sec \theta)^3 \sqrt{(2 \sec \theta)^2 - 4}}$$

$$= \int \frac{\cancel{2} \sec \theta \tan \theta}{\cancel{8} \sec^3 \theta \sqrt{4 \sec^2 \theta - 4}} \, d\theta$$

$$\text{let } x+1 = 2 \sec \theta$$

$$1 + 0 = 2 \sec \theta \tan \theta \frac{d\theta}{dx}$$

$$\boxed{dx = 2 \sec \theta \tan \theta \, d\theta}$$

$$\frac{x+1}{2} = \sec \theta$$

$$\cos \theta = \frac{2}{x+1}$$

$$\theta = \cos^{-1} \left(\frac{2}{x+1} \right)$$

$$\begin{aligned}
 & \int \frac{\tan \theta}{4 \sec^2 \theta \sqrt{4 \sec^2 \theta - 4}} d\theta \\
 &= \int \frac{\tan \theta}{4 \times 2 \sec^2 \theta \sqrt{\sec^2 \theta - 1}} d\theta \\
 &= \int \frac{\tan \theta}{8 \sec^2 \theta \sqrt{1 + \tan^2 \theta}} d\theta \\
 &= \int \frac{\cancel{\tan \theta}}{8 \sec^2 \theta \times \cancel{\tan \theta}} d\theta \\
 &= \frac{1}{8} \int \cos^2 \theta d\theta \\
 &= \frac{1}{8 \times 2} \int (\cos 2\theta + 1) d\theta
 \end{aligned}$$

$$\begin{aligned}
 \cos 2\theta &= 2\cos^2 \theta - 1 \\
 \cos 2\theta + 1 &= 2\cos^2 \theta
 \end{aligned}$$

$$\frac{\cos 2\theta + 1}{2} = \cos^2 \theta$$

$$\frac{1}{16} \left[\int \cos 2\theta d\theta + \int 1 \cdot d\theta \right]$$

$$\frac{1}{16} \left[\frac{\sin 2\theta}{2} + \theta \right]$$

$$\frac{1}{16} \left[\frac{\sin 2 \left[\cos^{-1} \left(\frac{2}{x+1} \right) \right]}{2} + \cos^{-1} \frac{2}{x+1} \right] + c$$

Ans

